Physics 319 Laboratory: Optics
Birefringence I

Objective: In this lab, the birefringence of calcite is to be studied. Some of this lab comes directly from Optics, Experiments and Demonstrations, C. Harvey Palmer, John Hopkins Press, Baltimore.

Apparatus: You will need two calcite crystals (one large and one small), a ruler, a protractor, a polarizer (from the Pasco optics kit), and a pair of vernier calipers.

Theory: A brief theory of birefringence is given in Hecht in section 8.4. It is strongly recommended that you read this section.

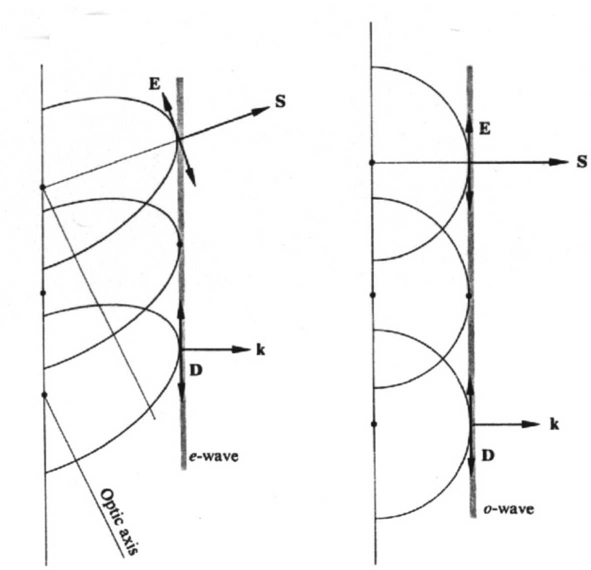
The electric field vector of light must have its orientation in the plane perpendicular to the direction of propagation. It is therefore convenient to define a coordinate system where one axis is parallel to the direction of propagation of the light. The components of the electric field vector along the other two axes define the two linear polarization states of the light.

A birefringent crystal has two indices of refraction. Waves polarized parallel to the "optical axis" of the material move at a velocity c/n_e and waves polarized perpendicular to the optical axis move at a velocity c/n_o. The subscript e stands for extraordinary and o stands for ordinary. The optical axis is a direction within the crystal defined by the molecular structure of the crystal.

When an arbitrarily polarized beam enters a birefringent crystal, it splits into two component beams, one polarized perpendicularly to the optical axis, called the ordinary ray (or o-ray), and one with its polarization in a plane which includes the optical axis, called the extraordinary ray (or e-ray). They travel independently in separate directions at different velocities.

The refraction of light entering a birefringent crystal does not obey Snell's law. On entering a crystal at normal incidence the o-ray continues through the crystal undeviated, but the e-ray deviates from a straight line path (in violation of Snell's law).

Consequently, when one views an object through a birefringent crystal, one sees two images - an image which remains still as the crystal is rotated (the o-ray) and an image which rotates with the crystal (the e-ray).



An explanation for this behavior can be sought in Huygens' principle. Consider waves striking the crystal at normal incidence and with polarization perpendicular to the optical axis and to the normal direction of the crystal (i.e. the ordinary ray)

In the right hand side of the figure above these waves are represented, they are incident from the right, the Huygens' wavelets are drawn on the transmitted side and the polarization vector would point out of the page. The Huygens wavelets spread out in uniform circles because all in possible directions of propagation with this polarization axis the waves will move at the same velocity, as in the right hand side of the figure.

By contrast, consider waves striking at normal incidence with polarization perpendicular to the normal of the surface, but within a plane that includes the optical axis crystal (i.e. the extraordinary ray). The velocity of propagation for the Huygens wavelets in the crystal for these waves will depend on the direction of propagation. Those traveling along the optical axis are polarized perpendicularly to the optical axis, and consequently travel at c/n_o, the one traveling perpendicularly to the optical axis must be polarized along the optical axis, and therefore travel at c/n_e . Consequently the Huygens wavelets are not circular in shape, but elliptical, since they travel faster in the direction perpendicular to the optical axis (In Calcite, as indicated in the figure above, the waves polarized along the optical axis travel fastest). See the left hand side of the above figure.

The direction of propagation of the energy is defined by the envelope of the Huygens wavelets as they expand; each point on the earlier wavefront determines just one point on the envelope of elliptical wavelets. A vector labeled S connecting these two points on the figure indicates the direction of energy transport.

Procedure:

I. Shape of the calcite crystal

Examine the shape of the calcite crystals provided. The crystal can be cleaved in the directions parallel to the natural faces. How many kinds of corners are there? Describe each variety of corner in terms of the number of acute angles and obtuse angles made by the three edges. On a sheet of paper, lay both the small crystal and a ruler. Hold the crystal to the paper so that it does not slide and place the ruler so that one edge makes contact with one face of the crystal. Draw a line about 6 inches long using the other edge of the ruler (i.e., parallel to the crystal edge). Repeat for the other three edges so as to obtain a parallelogram whose dimensions are those of the crystal plus two widths of the ruler. Measure and label the angles of the resulting parallelogram.
Make a three-dimensional sketch of the crystal.

II. Birefringence

Put a dot on a piece of paper and place a crystal over it. Sketch the top face of the crystal and put one dot in the center of the face. Carefully indicate the position of the second dot, observing both the angle with respect to the first and the separation (the observer is assumed to be looking vertically down, normal to the crystal face).

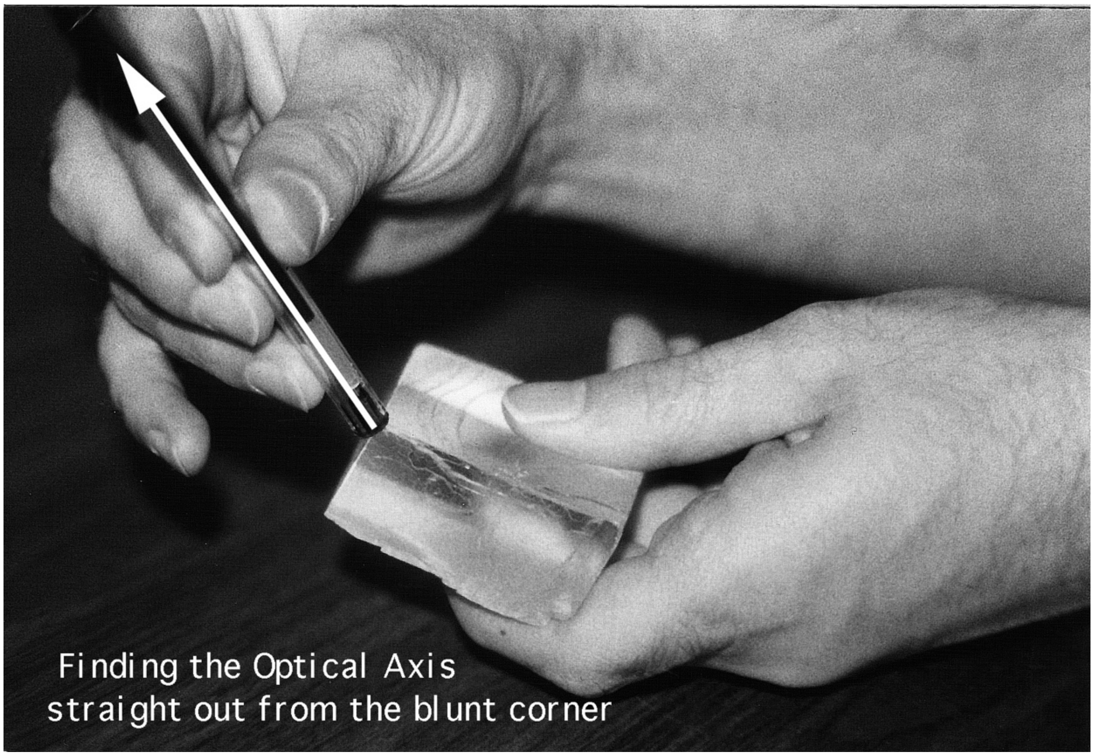
Identify which dot corresponds to the o-ray and which to the e-ray. To identify the two, rotate the crystal and observe which dot moves. Use a polarizer from the optics kit to identify the direction of polarization of each dot, and indicate the polarizations in your sketch by means of a short dash through each dot. Identify each of the dots as 0 or E.

Which dot appears to be deeper? What do your observations indicate about the relative indices of refraction for the o- and e-rays? Verify that the separation of the dots both in apparent depth and in horizontal separation increases linearly with the thickness of the crystal.

III. Angle of deviation of e-ray

Using the ruler, draw two lines forming a V about 7 inches long and 1/4 of an inch wide at the top. Lay the crystal on the lines and orient it to give maximum separation of the lines. Slide the crystal along the lines and ascertain at what point two of the four lines merge so as two give three in all. Measure accurately the separation of the lines at this point, and measure the thickness of the crystal with the vernier calipers. Repeat the measurements 5 times on the same crystal, recording each measurement, then averaging. Calculate the angle of deviation of the e-ray. (Hint: you have the lengths of two sides of a right triangle, and you need to calculate an angle. Recall your trigonometry.) Average these values to obtain the angle of deviation of the e-ray for calcite. Compare your results with theoretical predictions of Kroeger:
http://www.olemiss.edu/courses/phys319/Calcite.

IV. The Optical Axis



The optical axis can be identified by finding the blunt corner, one where three 109 degree angles come together in one corner. The optical axis points straight out from this corner in such a way that the three faces have a three fold symmetry about the optical axis. Find the blunt corner and hold a pencil in the direction of the optical axis, rotate the crystal until this axis is pointing up and away from you. Observe the alignment of the two images relative to this axis.
Now bring up a polarizer and rotate to find the orientation relative to the optical axis which extinguishes the ordinary ray, then the extraordinary ray.

Post Lab Questions- (3/21/21 version)

1. Does the extraordinary ray get extinguished when the polarization axis is parallel to or perpendicular to the optical axis? **(Questions are continued on next page)**

2. A) Is the extraordinary ray displaced toward or away from the blunt corner, relative to the ordinary ray. B) What is the direction (orientation) of the Poynting vector **S** to the optical axis. C) Is the ordinary ray displaced at all? **See figure 8.23 of text or 1st figure in lab writeup.**

3. The difference $\Delta n = (n_e - n_o)$ is called the birefringence. This difference defines two types of crystals (negative and positive uniaxial).

   A) what type is calcite? See question below for indices of refraction.

   B) Looking at figure 8.23 again- Is this the figure correct for calcite (i.e., are the ratios of semi-minor axis to semi-major (of the ellipses) correct for the ratio of indices of refraction for calcite? We can easily show that $n_e/n_o = v_o/v_e$.
   You will need a ruler to measure lengths. Show all work and lengths.

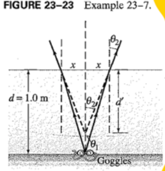
   c) Redraw (sketch by hand is fine) the Huygens' wavelet shown in figure 8.24 with the semi-minor to semi-major ration equal to ratio of indices of refraction.

4. A) Which dot looks deeper than the other. Why does one dot appear deeper in the crystal than the other? See explanation below. Figure & example taken from **Physics** by Giancoli.

   B) Using the circled equation draw two diagrams that look like the diagram below. One diagram using index of refraction of ordinary ray **(1.6584)** & the 2nd using index of refraction of extraordinary ray **(1.4864).**

   Assume thickness is the thickness that you measured for your crystal. Draw to scale. Show all work.

FIGURE 23–23 Example 23–7.



EXAMPLE 23–7 Apparent depth of a pool. A swimmer has dropped her goggles to the bottom of a pool at the shallow end, marked as 1.0 m deep. But the goggles don't look that deep. Why? How deep do the goggles appear to be when you look straight down into the water?

APPROACH We draw a ray diagram showing two rays going upward from a point on the goggles at a small angle, and being refracted at the water's (flat) surface. This is shown in Fig. 23–23, and the dashed lines show why the water seems less deep than it actually is. The two rays traveling upward from the goggles are refracted *away* from the normal as they exit the water, and so appear to be diverging from a point above the goggles (dashed lines).

SOLUTION To calculate the apparent depth d' (Fig. 23–23), given a real depth $d = 1.0$ m, we use Snell's law with $n_1 = 1.33$ for water and $n_2 = 1$ for air:

$$\sin\theta_2 = n_1 \sin\theta_1 .$$

We are considering only small angles, so $\sin\theta \approx \tan\theta \approx \theta$, with θ in radians. So Snell's law becomes

$$\theta_2 \approx n_1\theta_1 .$$

From Fig. 23–23, we see that

$$\theta_2 \approx \tan\theta_2 = \frac{x}{d'} \quad \text{and} \quad \theta_1 \approx \tan\theta_1 = \frac{x}{d}.$$

Putting these into Snell's law, $\theta_2 \approx n_1\theta_1$, we get

$$\frac{x}{d'} \approx n_1 \frac{x}{d}$$

or

$$d' \approx \frac{d}{n_1} = \frac{1.0\text{ m}}{1.33} = 0.75\text{ m}.$$

The pool seems only three-fourths as deep as it actually is.

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